Lec (7)

ch3 تابع

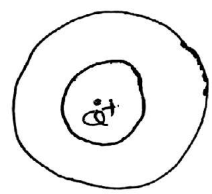
1- Electric Field due to a shell at which There is a point charge at its Center.

2- A charged Conducting shell with charge Q & Point charge at its Center (q)

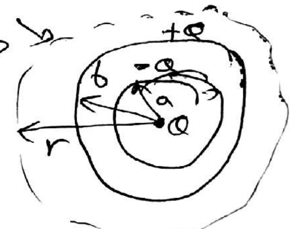
3- Insulated Shell

[1] E due to a shell at which there is a point charge at its Center

كرة موصلة مفرغة وفي مركزها شحنة نقطية (shell غير مشحونة بس في مركزه شحنة)



تقول القاعدة أن الشحنة +Q اللي في المركز تولد عن طريق الحث inductance شحنة مساوية لها في المقدار ومضادة لها في الإشارة على السطح الداخلي وتولد شحنة أخرى على السطح الخارجي مساوية في المقدار والإشارة

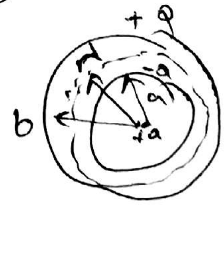


(i) $r > b$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q$$

$$\varepsilon_0 \oint E \cdot dA = Q - Q + Q$$

$$\varepsilon_0 E \, 4\pi r^2 = Q \quad \therefore \quad E = \frac{Q}{4\pi \varepsilon_0 r^2}$$



(ii) $a < r < b$

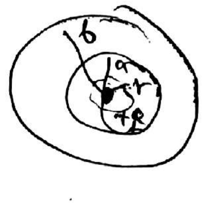
$$\varepsilon_0 E \, 4\pi r^2 = -Q + Q \quad \text{zero}$$

$$\therefore E = 0$$

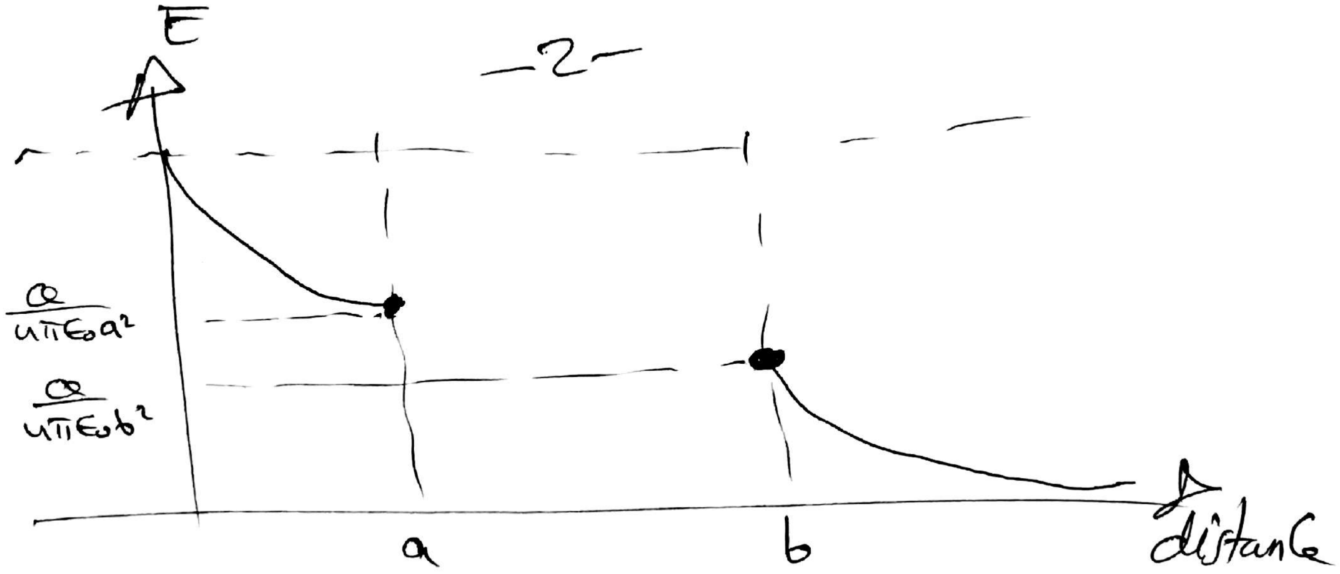
(iii) $r < a$

$$E \varepsilon_0 4\pi r^2 = +Q$$

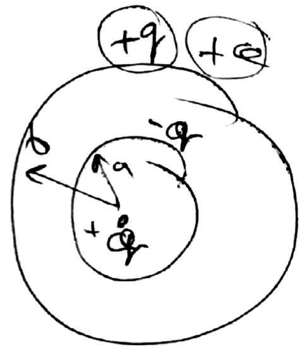
هنا فيه شحنة المركز بس



$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$



2 A charged Conducting shell with charge "Q" & Point charge "q" at Center



① E r < a (+q only)

$\therefore \epsilon_0 E(4\pi r^2) = q$

$E = \frac{q}{4\pi\epsilon_0 r^2}$

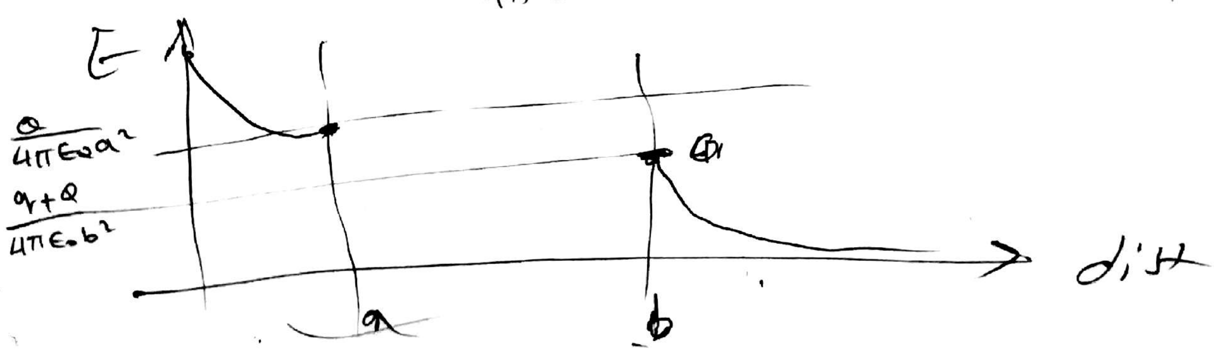
② E b > r > a

$\epsilon_0 (4\pi r^2) = q - q = zero$

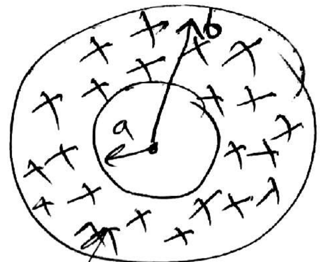
$\therefore E = 0$

③ r > b $\epsilon_0 E(4\pi r^2) = (q+Q) - q + q$

$\therefore E = \frac{q+Q}{4\pi\epsilon_0 r^2}$



[3] insulated shell



مفيش شحنة جوه

(i) $E|_{r<a}$ (no charge)

$\therefore \varepsilon_0 E(4\pi r^2) = 0$

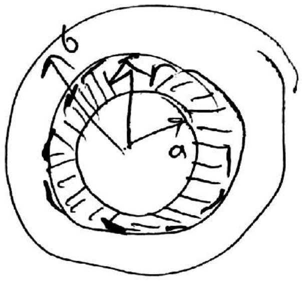
$\therefore E = 0$

ii) $E|_{r>b}$

$\varepsilon_0 E(4\pi r^2) = Q_{en}$ ⟶ $\rho_v * V$; $V \to \frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3$ (حجم المظلل)

$\therefore E = \dfrac{Q_{en}}{4\pi\varepsilon_0 r^2}$

iii) $E|_{b>r>a}$



$\varepsilon_0 E(4\pi r^2) = Q'$

$Q' \Rightarrow ??$ $\rho_v = \dfrac{Q'}{V}$; $Q' = \rho_v \cdot V$

$Q' = \rho_v \left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right]$

$\therefore E = \dfrac{Q'}{4\pi\varepsilon_0 r^2} = \dfrac{\rho_v \left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right]}{4\pi\varepsilon_0 r^2}$

$$E = \frac{\rho_v (r^3 - a^3)}{3\varepsilon_0 r^2}$$

note [D] electric flux density ($\bar{D}$) (vector) $= \epsilon \bar{E}$

$\psi = \oint \bar{D} \cdot d\bar{s} = Q_{en}$

Sheet (4)

أول D مسائل Report (آخر شيت)

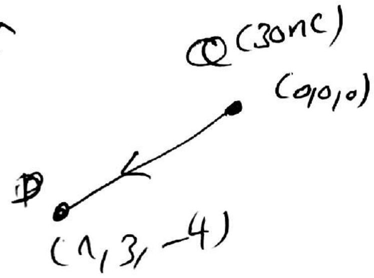
يتم تسليم مسائل (شيت ١ (٢)
شيت ٢ (٢)
شيت ٣ (١)
شيت ٤ ٥

مع موعد امتحان ٧/٢٢

sheet (4 - prob 6)

A point charge $Q = 30nC$, is located at the origin in Cartesian coordinates. Find the Electric Flux density D at (1, 3, -4)

sol



$$\bar{D} = \epsilon \bar{E} = \epsilon \cdot \frac{Q}{4\pi\epsilon r^2}\hat{a_r} = \frac{Q}{4\pi r^2}\hat{a_r}$$

$$= \frac{Q}{4\pi r^3}\bar{R}$$

$$\bar{R} = (1-0, 3-0, -4-0) = \hat{a_x} + 3\hat{a_y} - 4\hat{a_z}$$

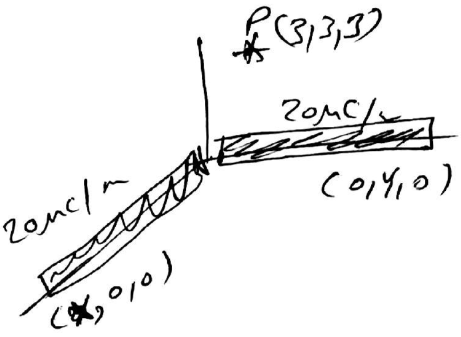
$$|\bar{R}| = \sqrt{(1)^2 + (3)^2 + (-4)^2} = \sqrt{26}$$

$$\therefore \bar{D} = \frac{30\times10^{-9}}{4\pi(\sqrt{26})^3} * (\hat{a_x} + 3\hat{a_y} - 4\hat{a_z})$$

Sheet (4) - Prob (7)

Two identical uniform line charges lie along x and y axes with $\rho_L = 20\mu C/m$. obtain $\bar{D}$ and $\bar{E}$ at (3, 3, 3) m

Sol



$\bar{D}_{tot} = \bar{D}_1 + \bar{D}_2 = \epsilon \bar{E}_1 + \epsilon \bar{E}_2$

$D_1 = \epsilon \bar{E}_1 = \frac{\rho_L}{2\pi r_1} \hat{a}_{r1} = \frac{\rho_L}{2\pi r_1^2} \bar{R}_1$

$\bar{R}_1 = (3-3, 3-0, 3-0)$

$\hookrightarrow_{x=3} = 0\hat{a}_x + 3\hat{a}_y + 3\hat{a}_z$

$|\bar{R}_1| = \sqrt{18}$

$D_2 = \frac{\rho_L}{2\pi r_2^2} \bar{R}_2$

$\bar{R}_2 = (3-0, 3-3, 3-0) = 3\hat{a}_x + 3\hat{a}_z$

$|\bar{R}_2| = \sqrt{18}$

$D_{tot} = \bar{D}_1 + \bar{D}_2 = \frac{20\times 10^{-6}}{2\pi} \left[\frac{\bar{R}_1}{r_1^2} + \frac{\bar{R}_2}{r_2^2} \right]$

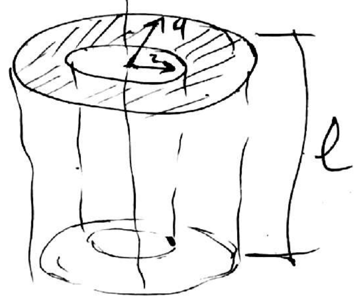
$D_{tot} = \frac{20\times 10^{-6}}{2\pi(\sqrt{18})^2} \left[3\hat{a}_x + 3\hat{a}_y + 6\hat{a}_z \right]$

$E_{tot} = \frac{D}{\epsilon}$ ✓

Sheet (4- Prob 8)

The volume in cylindrical coordinates between $r = 2m$ & $r = 4m$ contains a uniform charge density $\rho_v \ \mu C/m^3$. Use gauss's law to find D in all region.

الحل



① $r < 2$ $\qquad 4 > r \ \& \ 2 > r$ صح لأن الشحنة

$$q = \varepsilon_0 \oint E \cdot dA = \varepsilon_0 E (\pi r^2) = zero$$

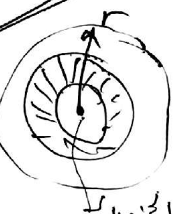
② $r > 4m$

$$q = \varepsilon_0 \oint E \cdot dA = \varepsilon_0 E (2\pi r * L) = \rho_v * V$$

$$\pi l (4^2 - 2^2)$$

$$V = \iiint dv = \int_{r=2}^{4} \int_{0}^{2\pi} \int_{0}^{l} r \, dr \, d\phi \, dz = \left[\frac{r^2}{2}\right]_2^4 [\phi]_0^{2\pi} [z]_0^l$$

$$= 2\pi l (8 - 2) = 12 \pi l$$



الجزء المفرغ هو الحجم

$\pi (4)^2 l - \pi (2)^2 l = 12 \pi l$

$$\varepsilon_0 E (2\pi r l) = \rho_v (12 \pi l)$$

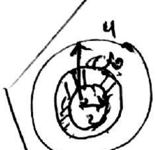
$$E = \frac{6 \rho_v}{\varepsilon_0 r}, \quad D = \frac{6 \rho_v}{r} a_r$$

③ $2 \le r \le 4$

$$\varepsilon_0 \oint E \cdot dA = \varepsilon_0 E (2\pi r l) = \rho [V_r - V_2]$$

$$= \rho_v [\pi r^2 l - \pi (2)^2 l]$$

$$\varepsilon_0 E (2\pi r l) = \rho_v \pi l (r^2 - 4)$$



$\pi (r)^2 l - \pi (2)^2 l$

$$E = \frac{\rho_v (r^2 - 4)}{2 \varepsilon_0 r}, \quad D = \frac{\rho_v (r^2 - 4)}{2r} a_r$$

Sheet (4) - prob (9)

The volume in spherical coordinate has radius r=a, and contains a uniform charge density ρ_v $\mu C/m^3$. Use Gauss's law to find D. What point charge at origin will result in the same D field for r>a?

$$q = \varepsilon_0 \oint E \cdot dA = \varepsilon_0 E (4\pi r^2) \Rightarrow \rho_v * V, \quad V = \frac{4}{3}\pi a^3$$

$$\therefore E = \frac{\rho_v a^3}{3\varepsilon_0 r^2} \qquad D = \frac{\rho_v a^3}{3r^2} \hat{a_r}$$

1) at r=a : $D = \frac{\rho_v a}{3} \hat{a_r}$ اهم حاجة في المسألة

2) Point charge : $D = \frac{Q}{4\pi r^2} \hat{a_r}$

$$\therefore \text{let } \frac{\rho_v a^3}{3r^2} = \frac{Q}{4\pi r^2}$$

$$\therefore Q = \frac{4}{3}\rho_v a^3$$